Generalization of the sheath criterion in an anisotropic plasma

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Abstract. Stangeby and Allen have demonstrated that the plasma–sheath boundary is a Mach surface in a collisionless plasma containing an anisotropic cold ion velocity distribution and no ionization. It is shown that the plasma–sheath boundary is still a Mach surface even when the effects of ionization, momentum-transferring collisions and a finite ion temperature are considered.

In a recent publication (Stangeby and Allen 1970) it has been demonstrated that for collisionless, ionization-free, zero-temperature ion motion, the plasma–sheath boundary may be identified as a closed Mach surface, thus demonstrating that the Bohm criterion applies to the component of ion velocity normal to the sheath, independent of the ion velocity component tangential to the sheath. In fact, the analysis may be extended to include ionization and collisional ion motion, i.e. to the case of intermediate pressures.

Considering a neutral plasma in which the number density of ions or electrons is N, and assuming that the ionization rate per unit volume is proportional to N, the steady-state continuity and momentum equations for the ions are

$$\nabla \cdot (N\boldsymbol{q}) = \lambda N \tag{1}$$

and

$$M\frac{\mathrm{d}\boldsymbol{q}}{\mathrm{d}t} = -e\nabla V - \frac{kT_{\mathrm{i}}}{N}\nabla N - M(\nu+\lambda)\boldsymbol{q} \tag{2}$$

where λ is the ionization rate per electron, ν is the ion–neutral collision frequency for momentum transfer, V is the electrostatic potential, T_{i} is the ion temperature and e and M are the ionic charge and mass, respectively. The system of equations is closed by the Boltzmann density expression

$$N = N_0 \exp(eV/kT_{\mathrm{e}}) \tag{3}$$

where $N = N_0$ when $V = 0$ and T_{e} is the electron temperature, together with the assumption of irrotationality (see Woods 1965):

$$\nabla \times \boldsymbol{q} = 0. \tag{4}$$

Using curvilinear coordinates (n, s) where n and s are the normal and transverse coordinates for any simple contour C, and eliminating N and V from equations (1)–(4), we obtain

$$\frac{\partial q_n}{\partial n}\left(1-\frac{q_n^2}{a^2}\right) + \frac{q_n}{R(s)}\left(1+\frac{q_s^2}{a^2}\right) + \frac{\partial q_s}{\partial s}\left(1-\frac{q_s^2}{a^2}\right) - \frac{2q_nq_s}{a^2}\frac{\partial q_n}{\partial s} - \lambda - (\nu+\lambda)\frac{(q_n^2+q_s^2)}{a^2} = 0 \tag{5}$$

where $a = \{k(T_{\mathrm{e}}+T_{\mathrm{i}})/M\}^{1/2}$ is the ion speed of sound and $R(s)$ is the radius of curvature of C at s. (If the charged particle behaviour is adiabatic, then the Boltzmann relation for the electrons is replaced by the continuity and momentum equations for the electrons, and the two-fluid equations combined resulting in no essential alteration to the above analysis except that a becomes $\{(\gamma_{\mathrm{e}}kT_{\mathrm{e}}+\gamma_{\mathrm{i}}kT_{\mathrm{i}})/(M_{\mathrm{i}}+M_{\mathrm{e}})\}^{1/2}$, where γ_{e} and γ_{i} are the specific heat ratios for the electrons and ions, respectively. The isothermal case is regained by setting $\gamma_{\mathrm{e}} = \gamma_{\mathrm{i}} = 1$). Equation (5) is equivalent to equation (13) in Stangeby and Allen's (1970) paper. As before, we choose C to be a closed Mach line C_M (so that $q_n = -a$ and $\partial q_n/\partial s = 0$ on C_M) and integrate around C_M, yielding

$$\lim_{q_n \to -a} \oint_{C_M} \frac{(1-q_n^2/a^2)}{(1+q_s^2/a^2)} \frac{\partial q_n}{\partial n} ds = 2\pi a + \lambda \oint_{C_M} \frac{ds}{(1+q_s^2/a^2)} + (\nu+\lambda) \oint_{C_M} ds. \qquad (6)$$

In general, the right-hand side of equation (6) is positive, so that

$$\lim_{q_n \to -a} \oint_{C_M} \frac{(1-q_n^2/a^2)}{(1+q_s^2/a^2)} \frac{\partial q_n}{\partial n} ds > 0$$

and $\partial q_n/\partial n \to \infty$ on at least some portion of C_M. The analysis of Stangeby and Allen (1970) may now be applied to show that $\partial q_n/\partial n \to \infty$ everywhere on C_M.

Hence, the plasma–sheath boundary in a plasma with an anisotropic ion velocity distribution is a Mach line, i.e. the normal component of the ion velocity is equal to $\{k(T_e + T_i)/M\}^{1/2}$ irrespective of ionization or collision effects in the plasma.

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A possible mechanism for instability in a perpendicular collisionless shock wave

Abstract. Bernstein waves which propagate in a direction opposite to the current flow in a perpendicular collisionless shock can have negative energy. These negative-energy waves can give rise to instability either by coming into resonance with the ion acoustic wave or by dissipating their energy through ion Landau damping. In the second case the instability can take place for $T_i \gtrsim T_e$.

There has been much speculation about the nature of the instability which might occur within a collisionless shock wave propagating perpendicular to a strong magnetic field. Sagdeev (1966) has suggested an ion wave instability and this idea has recently been refined by Krall and Book (1969). Krall and Book considered waves propagating perpendicular to the magnetic field $\boldsymbol{B}_0$ and the shock front, which were driven unstable by the perpendicular drifts due to the gradients of magnetic field and density at the front. However, Gary and Sanderson (1970) have pointed out that there is a third source of drift, due to the voltage jump across the shock front. For low or moderate values of $\beta_e(\beta_e = n_0\kappa T_e/(B_0^2/2\mu_0))$ the drift due to the voltage jump is the dominant one (Gary and Sanderson 1970).